Oil Shocks and Macroeconomic Activity: a Putty-Clay Perspective

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Abstract

I extend the Atkeson and Kehoe (1999) putty-clay model to include elastic labor supply and more general forms of technology to explore the impact of oil shocks on the macroeconomy. In particular, I am interested in (1) how this extension affects their results with regard to permanent changes in the price of oil, (2) a comparison of the business cycle properties of the putty-putty and putty-clay models, and (3) whether or not this extended putty-clay model is subject to the Rotemberg and Woodford (1996) critique of the standard perfectly competitive real business cycle model with energy. Results are reported for a wide range of parameter values illustrating that (1) contrary to the Atkeson-Kehoe result, the response of output and capital to permanent changes in the price of oil is identical in both the putty-putty and putty-clay models and is sensitive to the elasticity of substitution between capital services and labor, (2) there are stark differences in several business cycle features, namely the volatility of energy use and the correlations of output with consumption, investment and hours, and (3) the Rotemberg-Woodford critique applies to the putty-clay model revealing both amplification and timing problems.

Keywords: energy, putty-clay, dynamic general equilibrium.
JEL: E32 and Q43.

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1 Introduction

Changes in the price of oil affect the macroeconomy. This claim qualifies as a well-documented empirical fact in macroeconomics, but by itself is not very useful information. More important are how exactly and why? The oil-macroeconomy relationship has an extensive empirical and theoretical literature bent on providing answers to these questions. Dynamic general equilibrium models have been used to better understand the oil-macroeconomy relationship. At least three modeling assumptions have proven to be important: (1) are markets perfectly competitive, (2) is capital fully utilized, and (3) how do capital and energy combine to produce capital services, i.e., should capital be modeled as putty-putty or putty-clay? Criteria for assessing these models include both short-run effects (business cycles properties and impulse response functions to temporary changes in the price of oil) and long-run effects (steady-state effects of permanent changes in the price of oil).

Kim and Loungani (1992) explore the business cycle properties of a standard real business cycle model using a putty-putty technology with interest in discovering the extent to which the introduction of oil shocks reduces the model’s reliance on technology shocks to generate business cycles quantitatively similar to those in the U.S. They also document two business cycle facts: (1) energy use is slightly more volatile than output and (2) energy prices and output are negatively correlated (-0.44). They find that energy shocks alone can explain about 1/3 of the volatility of output. Their model is capable of reproducing the negative correlation between energy prices and output, but grossly over predicts the volatility of energy use (almost 4 times that of output). Rotemberg and Woodford (1996) estimate impulse response functions illustrating the effect of an exogenous change in the nominal price of oil’s growth rate on output and wages. They convincingly argue that the perfectly competitive real business cycle model is incapable of replicating these responses, but offer a model with imperfect competition that can. However, Finn (2000) constructs a perfectly competitive model similar to Kim and Loungani (1992), but with variable capital utilization that is capable of reproducing the estimated responses of Rotemberg and Woodford.

The papers above all treat capital as homogeneous and the relationship between capital and energy as putty-putty. However, Atkeson and Kehoe (1994, 1999) use a putty-clay technology for energy use. Through several judicious simplifying assumptions, Atkeson and Kehoe are able to side-step the insolvability problem of the putty-clay model with an aggregation result that focuses on aggregate capital services (the valued added of capital when combined with energy) as opposed to the differentiated capital stock itself. Consequently, the equilibrium in their model can be solved using only the level of capital services with no need to keep track of the specific amounts of the potentially thousands of types of capital. Atkeson and Kehoe find that both the putty-putty and putty-clay model have similar implications for energy use in the time series and cross section to temporary changes in the price of oil. But with regard to permanent changes in the price of oil, they report that the putty-putty model grossly over predicts the impact on capital and output (by an order of magnitude), whereas the putty-clay model delivers predictions similar to those estimated in the data. Furthermore, Atkeson and Kehoe (1994) uncover a qualitative asymmetric re-

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response to positive and negative oil shocks whereas the putty-putty model gives symmetric responses.\textsuperscript{2}

In this paper, I extend the putty-clay model of Atkeson and Kehoe (1999) to include elastic labor supply and more general forms of technology to explore the impact of oil shocks on the macroeconomy. In particular, I am interested in the following questions: (1) how these extensions affect their results with regard to permanent changes in the price of oil, (2) a comparison of the business cycle properties of the putty-putty and putty-clay models, and (3) whether or not this extended putty-clay model is subject to the Rotemberg and Woodford (1996) critique. The extended putty-clay model is calibrated and numerically simulated. Results are reported for a wide range of parameter values illustrating: (1) contrary to the Atkeson-Kehoe result, the response of output and capital to permanent changes in the price of oil is identical in both the putty-putty and putty-clay models and is sensitive to the elasticity of substitution between capital services and labor, (2) there are stark differences in several business cycle features, namely the volatility of energy use and output and the correlations of output with consumption, investment and hours, and (3) the Rotemberg-Woodford critique applies to the putty-clay model illuminating both amplification and timing problems.

I describe the model in section 2, provide details on the model calibration and report the numerical results in section 3, and conclude the paper in section 4.

2 Model

In this section, I describe the extended Atkeson-Kehoe model with particular attention to the putty-clay technology. Value added is produced with a production function that depends on capital services and labor. Capital services is produced by combining capital with energy (e.g., putting fuel in to the engine or plugging the machine into a power outlet). It is natural to think of a unit of capital as having a maximum level of capital services it can provide, and the production of capital services (up to the maximum level) is constant returns to scale in energy use (plugging the machine in for 6 hours gives twice as much capital services as plugging it in for 3 hours).\textsuperscript{3} Given that energy is a costly factor input, all else equal, firms would like to use more energy efficient types of capital. However all things are not equal, and in some sense obtaining more efficient capital must be more costly. This is accomplished by making the maximum level of capital services from a unit of investment a decreasing function of the level of efficiency.

Types of capital are indexed by $v \in V$ (a possibly infinite set), where $v$ represent the energy intensity of production. Capital services are produced when energy is combined with capital. Combining $k_v$ units of type $v$ capital with energy $e$ delivers capital services according to the following

\[
\min \left( \frac{k_v}{v}, e \right) f(v),
\]

\textsuperscript{2}The asymmetric response of the macroeconomy to oil shocks is another well-documented fact, i.e., oil price increases have larger effects than oil price decreases. See Mork (1989), Hamilton (2003), and Davis and Haltiwanger (2001).

\textsuperscript{3}This treats utilization measured in time as the only margin – running the machine for twice as long requires twice the energy. Alternatively, one could think of running the machine for the same time, but at twice the speed. In this instance there is no reason to expect energy use to double.
where $f(v) > 0$, $f'(v) \geq 0$, and $f''(v) < 0$ for $v \in V$. Note that $k_v f(v)/v$ is the maximum level of capital services obtainable and this requires $e = k_v/v$ units of energy. Additional energy use beyond this level is wasteful. For $0 \leq e < k_v/v$, the marginal service of energy is given by $f(v)$. Since $f' > 0$, it is clear that a higher $v$ reflects more energy efficiency. However, since $f'' < 0$, we have $f(v)/v$ decreasing in $v$ so the more efficient types also have lower maximum capacity for capital services (recall it is this aspect that makes the more efficient types of capital less attractive). This trade-off is illustrated in Figure 1 where capital services as a function of energy use is plotted for two types of capital $v_1 < v_2$.

Figure 1: This figure illustrates the trade-off between an inefficient type of capital $v_1$ and an efficient type of capital $v_2 > v_1$.

Let $k_t(v)$ be the amount of capital of type $v$ at time $t$ and $e_t(v)$ be the energy used with this type of capital at time $t$. Aggregate capital services over all types of capital is given by

$$z_t = \int_V \min[k_t(v)/v, e_t(v)] f(v) dv.$$

Capital services $z_t$ are combined with labor $l_t$ to deliver value-added according to:

$$y_t := G(z_t, l_t) - p_t m_t,$$

where $m_t := \int_V e_t(v) dv$ is aggregate energy use and $p_t$ is the price of energy. The price of energy $p_t$ is exogenous and follows an ARMA(1,1) process:

$$\log p_{t+1} = (1 - \rho) \log \bar{p} + \rho \log p_t + \eta \epsilon_t + \epsilon_{t+1},$$

where $\epsilon_t$ is an iid mean zero process with standard deviation $\sigma$. Atkeson and Kehoe estimate $\rho = 0.9$, $\eta = 0.35$, $\bar{p} = 0.92$, and $\sigma = 0.108$.

All types of capital depreciate at the common rate $\delta$ and investment $x_t(v)$ must be nonnegative:

$$k_{t+1}(v) - (1 - \delta) k_t(v) = x_t(v) \geq 0.$$
The aggregate resource constraint is

\[ c_t + \int_V x_t(v)dv = y_t := G(z_t, l_t) - p_t m_t, \]

where \( c_t \) denotes aggregate consumption and \( y_t \) value added.

The investment decision can be thought of sequentially: (1) given a level of investment, what type of capital should be augmented, and (2) given the aggregate resource constraint, how much should be invested? To provide some intuition on the problem of choosing the type of capital in which to invest and changes in the price of oil on this decision, consider the following simpler problem. Suppose that one unit of investment is to be allocated between only two types of capital \( v_1 \) and \( v_2 \) with \( v_2 > v_1 \), and that value-added is given by \( \pi = G(z,l) - pm \). Under the assumption of full utilization, capital services is given by: \( z = kf(v)/v \) with \( e = k/v \). One more unit of investment in \( v_i \) increases \( z_i \) by \( f(v_i)/v_i \) and the cost of energy by \( p/v_i \), so the change in value-added is \( G_z(z,l)f(v_i)/v_i - p/v_i \). The firm selects either \( v_1 \) or \( v_2 \) depending on which is more profitable. Graphing the value-added as a function of \( p \) for each type of capital, we see for low \( p \), the contribution from \( v_1 \) is greater. There is a price \( p^* \) where the firm is indifferent between the two types and for \( p > p^* \), type \( v_2 \) is strictly preferred. Note that the more efficient investment contributes less through capital services: \( G_z(z,l)f(v_2)/v_2 < G_z(z,l)f(v_1)/v_1 \). However, the cost as a function of \( p \) is increasing more rapidly for the less efficient capital: \( p/v_1 > p/v_2 \). This illustrates the intuitive notion that greater efficiency is desirable when the price of oil is high, and that more efficient types are not always desirable. Figure 2 illustrates this optimal choice.

Figure 2: This figures illustrates the value added from one unit of investment in either type \( v_1 \) or \( v_2 \) capital \( (v_2 > v_1) \) as a function of the price of oil \( p \). For low oil prices, the less efficient capital \( v_1 \) is more profitable.

The potential tractability problem of the putty-clay model comes from keeping track of the distribution of capital (amounts and types of capital) which leads to the so-called “curse of
dimensionality.” Atkeson and Kehoe circumvent this problem with a clever aggregation result that, under the assumption of full utilization of each type of capital, shows the equilibrium of the model can be solved using only aggregate capital services and aggregate energy use (no need to keep track of the distribution of capital).

3 Calibration and Results

I extend the Atkeson-Kehoe model to include a labor/leisure choice with preferences given by

$$\max_{E_0} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t).$$

(1)

For $U(c, l)$, we use

$$U(c_t, l_t) = \left[ \frac{c_t^\gamma (1 - l_t)^{1-\gamma} (1-\psi)}{1-\psi} \right]^{1-\psi}.$$

for $\psi(\neq 1) > 0$, where $c_t$ is consumption, $h_t := 1 - l_t$ is leisure, and $l_t$ is hours worked. When $\psi = 1$, $U(c_t, l_t) = \gamma \log(c_t) + (1-\gamma) \log(1-l_t)$. These preferences nest those used in Atkeson-Kehoe as a special case when $\gamma = 1$, $l_t = 1$, and $\psi \to 1$. Our baseline model sets $\beta = 0.96$, $\psi = 1$, and $\gamma = 0.375$ (to produce steady-state labor supply equal to $1/3$).

Their aggregation result easily extended to a model with a labor/leisure choice. The equilibrium in the model can then be found by solving the following social planner’s problem: maximize (1) by choice of \( \{c_t, x_t, v_t, l_t, z_{t+1}, m_{t+1}\}_{t=0}^{\infty} \), given \( \{z_0, m_0\} \) and \( \{p_t\}_{t=0}^{\infty} \), subject to:

$$c_t + x_t \leq G(z_t, l_t) - p_t m_t$$

(2)

$$x_t \geq 0$$

(3)

$$z_{t+1} \leq (1-\delta) z_t + x_t \frac{f(v_t)}{v_t}$$

(4)

$$m_{t+1} \geq (1-\delta) m_t + \frac{x_t}{v_t}$$

(5)

$$\log p_{t+1} = (1-\rho) \log \bar{p} + \rho \log p_t + \eta \epsilon_t + \epsilon_{t+1}.$$  

(6)

Equations (4) and (5) clearly illustrate the trade-off between the energy-efficiency of capital and the ability of capital to deliver capital services and output. As $v$ increases, $f(v_t)/v_t$ decreases in equation (4) diminishing the impact of $x_t$ on next period’s capital services $z_{t+1}$, but $1/v_t$ decreases so the impact on next period’s energy use $m_{t+1}$ in equation (5) is diminished.

In addition to adding leisure to the utility function, our technology specification generalizes Atkeson and Kehoe by using the following constant elasticity of substitution (CES) functional forms:

$$f(v) := \left[ w_f v^{1-a} + (1-w_f) \right]^{1/(1-a)},$$

(7)

$$G(z, l) := \left[ w_G z^{1-b} + (1-w_G) l^{1-b} \right]^{1/(1-b)}.$$  

(8)

The parameters $w_G$ and $w_f$ are set to match labor’s share (0.57) and energy’s share (0.043) in the data.\(^4\) This results in $w_f = 0.879$ and $w_G = 0.432$. In this model, the long-run

\(^4\)In steady state, one has

$$\nu^* = \left\{ \frac{w_f \beta p}{(1-w_f)(1-\beta(1-\delta))} \right\}^{1/a}.$$
elasticiies of energy use \((\sigma_{ep})\) and capital \((\sigma_{kp})\) with respect to the price of energy are

\[
\sigma_{ep} = -\left(\frac{1}{a}\right) \left[ \frac{w_f}{w_f + (1-w_f)v^{a-1}} \right] = -\left(\frac{1}{a}\right) \tilde{s}_K,
\]

\[
\sigma_{kp} = \left(\frac{1}{a}\right) \left[ \frac{(1-w_f)v^{a-1}}{w_f + (1-w_f)v^{a-1}} \right] = \left(\frac{1}{a}\right) \tilde{s}_E.
\]

Note these are steady-state equilibrium elasticities and derived from log-linearizing the steady-state equilibrium conditions.

The putty-putty model is that described by Atkeson and Kehoe. This model is similar to that used by Kim and Loungani (1992), but with capital adjustment costs and a more general production function for capital services. Capital evolves according to

\[
k_{t+1} = (1-\delta)k_t + x_t - \phi(x_t/k_t)k_t,
\]

where \(\phi(x/k) = (D/2)(x/k - \delta)^2\) is the adjustment cost. Capital services \(F\) is produced with a CES technology using capital \(k\) and energy \(e\):

\[
F(k,e) := [w_Fk^{1-a} + (1-w_F)e^{1-a}]^{1/(1-a)}.
\]

Output is produced with a CES technology using capital services \(F\) and labor \(l\):

\[
G(F,l) := [w_GF^{1-b} + (1-w_G)l^{1-b}]^{1/(1-b)}.
\]

The elasticity of substitution between capital and energy \(\rho_{ke} = 1/a\), and the elasticity of substitution between capital services and labor \(\rho_{Fl} = 1/b\). There are four parameters for this specification of technology: \(w_F\), \(w_G\), \(a\), and \(b\). Atkeson and Kehoe set these parameters to match market shares and the estimated short-run Allen elasticity of energy use with respect to energy price, \(\epsilon_{ep} = -1/3\), and the cross-steady-state Allen elasticity of energy use with respect to energy price, \(\eta_{ep} = -1\):

\[
\epsilon_{ep} = -\frac{1}{(1-s_L)a + \tilde{s}_E b/s_l}, \tag{11}
\]

\[
\eta_{ep} = -\frac{1 - \tilde{s}_E}{a} - \frac{s_L \tilde{s}_E}{b}, \tag{12}
\]

where \(s_L (=0.57)\) is labor’s share, \(s_E (=0.043)\) is energy’s share, \(s_K\) is capital’s share, \(\tilde{s}_E := s_E/(1-s_L)\) and \(\tilde{s}_K := s_K/(1-s_L)\) are energy’s and capital’s shares of the combination of capital and energy’s income. The implied values for \(a\) and \(b\) are \(a = 3.33\) and \(b = 0.079\). Kim and Loungani (1992) allow for CES between capital and energy, but specify a Cobb-Douglas between capital services and labor. A Cobb-Douglas specification corresponds to setting \(b = 1.001\). They consider two different values for \(a\): \(a = 1.001\) and \(a = 1.7\). These values for \(a\) are considerably lower than \(a = 3.33\) and their choice for \(b\) is considerably higher than

Using this expression, one can solve for steady state \(z^*\) as a function of \(w_f\) and \(w_g\). Steady state energy use is given by \(e^* = z^*/f(v^*)\). Using \(t^* = 1/3\), we have two equations \(pe^*/G(z^*,t^*) = S_E\) and \(G_l(z^*,t^*)t^*/G(z^*,t^*) = S_L\) in two unknowns \(w_f\) and \(w_g\).
 These differences implies drastically different impulse response functions and long-run responses to permanent changes in the price of oil. For \( a = 1.001 \), the short and long-run elasticities are \(-0.93\) and \(-0.96\). For \( a = 1.7 \), the short and long-run elasticities are \(-0.59\) and \(-0.59\). In this paper, in addition to the baseline \( a = b = 1.001 \), I consider two low \( \rho_{ke} \) economies \((a = 2.0 \text{ and } a = 3.0)\), two high \( \rho_{ke} \) economies \((a = 0.5 \text{ and } a = 0.1)\), one low \( \rho_{Fl} \) economy \((b = 2.0)\), two high \( \rho_{Fl} \) economies \((b = 0.5 \text{ and } b = 0.1)\).

There are three things I want to explore in this section. First, how does the addition of elastic labor supply and more general production technologies affect the model’s prediction for changes in output, capital, and energy use in response to permanent changes in the price of energy? Recall that the putty-clay model’s ability to match the estimated responses of these aggregates in the data as documented by Atkeson and Kehoe is one of the main advantages of this model over the putty-putty model. Second, what are the business cycle properties of the putty-clay model in comparison to the putty-putty model with energy of Kim and Loungani (1992)? Third, Rotemberg and Woodford (1996) claim the real business cycle model with perfect competition is incapable of generating large changes in both output and wages in response to changes in the price of oil of the magnitude measured in the data. Is the putty-clay model subject to this critique?

### 3.1 Energy Use, Output, and Capital

In this section, we explore the importance of elastic labor supply and the more general CES technologies on the impact of a permanent change in the price of oil on energy use, output, and capital. The impact on these variables is reported in Table 1 for both the putty-putty and putty-clay models for variety of technology parameters \( a \) and \( b \).

Changing the elasticity of labor supply had a negligible impact on these numbers.

\[ p = G_F F_e, \]
\[ 1 = \beta [G_F F_k + 1 - \delta]. \]

In percent changes we have

\[ [bs_K + (a - b)\bar{s}_K]\dot{k} + [bs_E + (a - b)\bar{s}_E - a]\dot{e} = \hat{p}, \]
\[ [bs_K + (a - b)\bar{s}_K - a]\dot{k} + [bs_E + (a - b)\bar{s}_E]\dot{e} = 0. \]

Solving for \( \dot{e} \) and \( \dot{k} \) gives

\[ \dot{e} = \left[ \frac{bs_K + (a - b)\bar{s}_K - a}{abs_L} \right] \hat{p}, \]
\[ \dot{k} = -\left[ \frac{\bar{s}_E(a - bs_L)}{abs_L} \right] \hat{p}, \]

\( ^5 \)Changing the elasticity of labor supply had a negligible impact on these numbers.
and substitution into the production function gives
\[
\hat{G} = s_K \hat{k} + s_E \hat{e} = - \left[ \frac{\hat{s}_E (1 - s_L)}{bs_L} \right] \hat{p}.
\]

The putty-clay model has the following steady-state conditions:
\[
v^a = pw_f \beta / [(1 - w_f)(\beta^{-1} - 1 + \delta)], \quad (17)
\]
\[
G_z f'(v) = \beta^{-1} - 1 + \delta, \quad (18)
\]
\[
z = mf(v). \quad (19)
\]

Equations (17) and (18) are derived from the intertemporal first-order conditions with respect to \( m \) and \( z \). In steady state there is one type of capital, so we have \( m = k/v, e = m, \) and equation (19), so capital services is simply \( z = ef(v) \) or equivalently \( z = kf(v)/v \). In percent deviations, we have
\[
\hat{v} = (1/a)\hat{p}, \quad (20)
\]
\[
\hat{z} = \hat{e} + \hat{s}_K \hat{v}, \quad (21)
\]
\[
0 = -bs_L \hat{z} - a \hat{s}_E \hat{v}. \quad (22)
\]

Solving for \( \hat{z} \) and \( \hat{e} \) gives
\[
\hat{z} = -(\hat{s}_E/bs_L)\hat{p}, \quad (23)
\]
\[
\hat{e} = - \left( \frac{a\hat{s}_E + b\hat{s}_K s_L}{abs_L} \right) \hat{p} = \left[ \frac{bs_K + (a - b)\hat{s}_K - a}{abs_L} \right] \hat{p},\quad (24)
\]
with the last equality coming from using \( \hat{s}_E = 1 - \hat{s}_K \) and \( \hat{s}_K s_L = \hat{s}_K s_k \). Using \( z = kf(v)/v \) implies
\[
\hat{z} = \hat{k} + \hat{f} - \hat{v} = \hat{k} + \hat{s}_K \hat{v} - \hat{v} = \hat{k} + (\hat{s}_K - 1)\hat{v} = \hat{k} - \hat{s}_E \hat{v}.
\]

Substituting for \( \hat{z} \) and \( \hat{v} \) gives
\[
\hat{k} = - \left[ \frac{\hat{s}_E(a - bs_L)}{abs_L} \right] \hat{p}.
\]

The total change in output is given by
\[
\hat{G} = - \left[ \frac{\hat{s}_E(1 - s_L)}{bs_L} \right] \hat{p}.
\]

One sees that these relationships are precisely the same relationship derived for the putty-putty model. The Cobb-Douglas case \((a = b = 1)\) implies the Atkeson-Kehoe result that \( \hat{G} = \hat{k} \).

In Table 1 it is evident that the elasticity of substitution between capital services and labor \( \rho_{zl} \) is the important parameter for amplifying the effect of doubling the price of oil on output and capital. Recall that Atkeson and Kehoe found the putty-putty model to greatly over estimate the impact of this change on output and capital in comparison to the
Table 1: Effect on output, capital, and energy use from the price of oil permanently doubling. Note the effects are essentially the same in both models.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>1.001</th>
<th>2.0</th>
<th>3.0</th>
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<td>1.001</td>
<td>1.001</td>
<td>1.001</td>
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<td>-10.90</td>
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<td>0.06</td>
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<td>-74.83</td>
<td>-99.89</td>
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<td>-79.42</td>
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putty-clay model. Their calibration set \( a = 3.33 \) and \( b = 0.079 \) for the putty-putty model, and \( a = b = 1.0 \) for the putty-clay. Recall that their choice for \( a \) and \( b \) was to match two elasticities of energy use with respect to the price of energy \( (\epsilon_{ep} = -1/3 \) and \( \eta_{ep} = -1) \).

In the putty-clay model, full capital utilization implies the short-run elasticity of energy use with respect to the price of energy \( (\epsilon_{ep} = 0) \). To construct a long-run elasticity of energy use with respect to the price of energy, I use the steady state conditions. The optimal change in \( v \) is given by \( a\tilde{v} = \hat{p} \). It is also true that \( \hat{f} = \tilde{s}_K\hat{v} \). Since in the long-run there is only one type of capital, \( z = ef(v) \) which implies \( \hat{z} = \hat{e} + \tilde{s}_K\hat{v} \). Substituting \( \hat{z} = 0 \) and \( \hat{v} = (1/a)\hat{p} \) into the previous expression yields \( \hat{e} = -(1/a)\tilde{s}_K\hat{p} \), giving us the long-run elasticity of energy use \( \sigma_{ep} = -(1/a)\tilde{s}_K \). If one wanted to calibrate the model so that \( \sigma_{ep} = -1 \) and \( \tilde{s}_K = 0.9 \), then setting \( a = 0.9 \) would accomplish this. Also, it is clear that since the calibration uses labor’s share and energy’s share, changes in \( b \) will have no effect on this long-run elasticity, leaving \( b \) as a free parameter not tied down by either the short-run or long-run elasticity of energy use with respect to the price of energy.

As a concluding remark in this subsection, note that the aggregate production function \( G(\cdot, \cdot) \) in both models captures how capital services (regardless of the way in which these capital services are produced) and labor combine to create output. On this ground, it is difficult to understand how \( b = 0.079 \) is admissible in one economy and \( b = 1.0 \) in the other.

### 3.2 Business Cycle Properties

In this section, I report business cycle properties of the putty-putty and putty-clay models. Standard deviations are reported in Table 2 and impulse response functions are reported in Figures 5–20. The most striking feature is the relatively high volatility in the putty-putty model. Energy use is much more volatile in the putty-putty model which leads to greater volatility in both capital services and output. The putty-clay model with full capital utilization is tying energy use to a rather slow-moving state variable the capital stock. In the both models, the lower elasticity of substitution between capital and energy (larger \( a \)), the lower the volatility in energy use, capital services, and output. For greater elasticities of
substitution between capital services and labor (smaller $b$) we get greater volatility in energy use, capital services, and output.

Correlations are reported in Table 3. In the Cobb-Douglas case ($a = b = 1$), the most notable distinction between the two models is the correlation between output and labor: $\rho(Y, l) = 0.94$ in the putty-putty model, and $\rho(Y, l) = -0.88$ in the putty-clay model. This difference can best be understood by looking at the impulse response functions for the baseline models in Figures 5 and 13 and considering the shifts in supply and demand in the labor market. In response to a positive shock to the price of oil, both models predict that there is a short-run decrease in consumption. This in turn increases the marginal utility of income and shifts the (Frisch) labor supply curve down, putting downward pressure on wages and upward pressure on equilibrium labor. This downward pressure on wages is reinforced with a leftward shift in the labor demand curve caused by the decrease in capital services. However, the upward pressure on equilibrium labor caused by labor supply is countered by the labor demand curve shifting left (the marginal product of labor diminishes as capital services decrease due to the decrease in energy use). As long as the decrease in capital services is sufficiently large, the net effect will be a decrease in equilibrium labor. Here the two models' quantitative differences matter. The decrease in capital services is much greater in the putty-putty model (larger swings in energy use) than in the putty-clay model. Consequently, the labor demand curve in the putty-clay model does not shift far enough to the left to result in a net decrease in equilibrium labor. Since swings in energy use and capital services is greater in the putty-putty model, the correlation between output and labor will tend to be greater (closer to 1).

3.3 Output and Wages

In this subsection, we want to see if the putty-clay model is subject to the Rotemberg and Woodford (1996) critique, namely that the perfectly competitive real business cycle model is not capable of matching the estimated response of output and wages to changes in the nominal price of oil growth rate. Using quarterly data (1947:2–1980:3), Rotemberg and Woodford estimate that a one percent shock to the growth rate of the nominal price of oil has an initial impact on real value added and value added real wages of 0.04 percent and 0.03 percent. The greatest decline in these two variables is approximately 0.25 and 0.09 percent occurring 5–6 quarters after the initial shock (near concurrent with the highest real price of oil following the shock). Their impulse response functions are shown in Figure 3. The putty-clay model is calibrated to closely follow Finn (2000) with one time period equal to a quarter: $\beta = 0.99$, $\delta = 0.025$, $\psi = 2$, $s_L = 0.7$, and $s_E = 0.043$. The real price of oil process is specified a part of a bivariate vector auto regression along with the growth rate of the nominal price of oil as in Rotemberg and Woodford (1996). As in Finn (2000), the model

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6. This occurs immediately in the putty-putty model, but only after the first period in the putty-clay model.

7. I experimented with different labor supply elasticities (different values for $\gamma$) and found very little difference. The impact was largest when capital services fluctuated the most or when the elasticity of substitution between labor and capital services was low (high $b$). These conditions imply larger movements in the demand for labor and so with a flatter labor supply curve will result in larger changes in equilibrium labor and output.
counterpart to the empirical data is to use a form of private value added that is meant to compensate for the domestic production of energy:

\[ G(z_t, l_t) - (1 - s_d)p e_t, \]

where \( s_d \) is the fraction of \( pe_t \) that is domestically produced.\(^8\) The private value added deflator is given by

\[ p^v_t = \frac{G(z_t, l_t) - (1 - s_d)p e_t}{G(z_t, l_t) - (1 - s_d)p e_t}. \]

Note that in percent deviations for the private value added deflator we have

\[ \hat{p}^v_t = -\frac{(1 - s_d)p e_t}{G(z, l) - (1 - s_d)p e_t}. \]

The private value added real wage in their data corresponds to

\[ w^v_t = \frac{w_t}{p^v_t}. \]

How does the putty-clay model compare?

One sees in Figure 4 that the putty-clay model suffers from both amplification and timing problems, and its impulse response functions are not consistent with those estimated by Rotemberg and Woodford. In the baseline model \((a = b = 1.001)\), the initial decline in private value added is about 0.01 percent, which is about 4 times too small. The greatest decline in real value added occurs about 15 quarters after the shock and is about 0.025 percent. This is 8 quarters too late and about 10 times too small. The initial change in the value added real wage is an \textit{increase} of 0.016 percent, which is \textit{qualitatively} incorrect (more on this later). The greatest decline in value added wages occurs about 23 quarters after the shock and is about 0.035 percent. This is 16 quarters too late and about 2.5 times too small. This result of “too little, too late” is borne out in other parameterizations as well. The best fit for output is when \( b \) is low (here the drop in output is largest about 0.09 percent and near the peak of the price impulse response function). The fit to the real real wage is best when \( a \) is low (here the drop about 0.04 percent and occurs around 21 periods after the price shock).

The source of the amplification and timing problem is easily traced to energy use. In the putty-clay model, energy use is not very volatile and reaches its low long after the the price of oil peaks. Why? With full capital utilization, energy use is essentially tied to the capital stock, a rather slow-moving state variable. Consequently, putty-clay energy use becomes a state variable whose propagation is similar to the capital stock (i.e., continues to falls even after the price of oil begins to return to its steady state value). Furthermore, the duration of falling energy use in response to a positive oil shock is prolonged by the choice of more efficient types of capital. This fact is most easily seen in the law of motion for energy use (in log-linear terms):

\[ \hat{m}_{t+1} = (1 - \delta)\hat{m}_t + \delta(\hat{x}_t - \hat{v}_t) = \hat{m}_t + \delta(\hat{x}_t - \hat{v}_t - \hat{m}_t), \]

\(8\)See Finn (2000) for more details on this mapping from model to data.
which reveals (1) the similarity to the evolution of capital equation, and (2) that energy use continues to fall as long as
\[ \hat{x}_t - \hat{v}_t < \hat{m}_t, \]
so the choice of more efficient types of capital (\( \hat{v} > 0 \)) propagates the oil shock. Whereas in the putty-putty model, energy use tracks the price process closely (energy use changes to maintain \( G_F F_e = p \)), so energy use, output, wages, and the price of oil all begin their return to steady-state values near concurrently.

Qualitatively, Rotemberg and Woodford (1996) find that the initial impact on real wages and real value added is negative. This qualitative response of both the real wage and real value added declining is not reproducible in the putty-clay model. Under the assumption of full capital utilization (a necessary condition for the Atkeson-Kehoe aggregation result), the level of capital services does not immediately change when the price of oil increases. Consequently in the labor market, changes in the real wage and equilibrium labor are movements along a fixed (downward-sloping) labor demand curve and so will move in opposite directions. For output to fall, the supply of labor must shift left causing the real wage to increase. Since the private value added deflator moves in the opposite direction of the price of oil, the reduction in the deflator coupled with the increase in the real wage results in the real value added wage \( w_v = w/p_v \) increasing. This implication of the putty-clay model is illustrated in Figure 4 by the initial increase in the real value added wage and decrease in real value added output.

Figure 3: Rotemberg and Woodford (1996) estimated impulse response functions for the real price of oil, real valued added, and the real wage in response to a one percent shock to the growth rate of the nominal price of oil along with one standard deviation error bands. See Rotemberg and Woodford (1996) for more detail.
Figure 4: Putty-Clay Model: impulse response functions of the wage and output to a one percent shock to the nominal price of oil.
4 Conclusion

In this paper, I have extended the Atkeson and Kehoe (1999) putty-clay model of energy use to include elastic labor supply and more general forms of technology for the production of capital services and output to explore the impact of oil shocks on macroeconomic activity. In particular, we were interested in (1) how this extension affects the model’s predictions for energy use, output and capital in response to permanent changes in the price of oil, (2) a comparison of the business cycle properties of the putty-putty and putty-clay models, and (3) whether or not this extended putty-clay model is subject to the Rotemberg and Woodford (1996) critique of the standard perfectly competitive real business cycle model with energy.

I find that the elasticity of substitution between capital services and labor is the key parameter in the model’s prediction of a permanent change in the price of oil on energy use, output, and capital. If the putty-putty and putty-clay technologies are the same, i.e., identical $F(\cdot, \cdot)$ and $G(\cdot, \cdot)$ functions, the two models have exactly the same prediction for the long-run impact on output, capital, and energy use in response to a permanent increase in the price of oil. The Atkeson and Kehoe conclusion that the putty-clay model performs better than the putty-putty model along this dimension is entirely due to their choice of elasticity of substitution of between capital services and labor (1 in the putty-clay model and about 12.66 for the putty-putty model). With regard to business cycle properties, the most striking feature is the relatively high volatility in the putty-putty model. Energy use is much more volatile in the putty-putty model which leads to greater volatility in both capital services and output. The putty-clay model’s predictions for energy use volatility are more in accord with the estimated volatility in Kim and Loungani (1992) than those of the putty-putty model. The Rotemberg and Woodford critique applies to the putty-clay model which suffers from amplification and timing problems. The model’s predictions for output and the real wage are “too little, too late.” In the baseline Cobb-Douglas economy, the largest decreases in the real wage is 4 times too small and output is 10 times too small with these maximum deviations occurring 6–7 quarters later than the estimated troughs in the data.
References


Table 2: Business Cycle Properties – Standard Deviations: Model statistics are averages over 5000 simulations of length 40 (each simulation is HP filtered). The standard deviations across the 5000 simulations are reported in parentheses. For the price of oil we have $\sigma(p) = 13.70$ (2.74).

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Table 3: Business Cycle Properties – Correlations with Output ($Y$): Model statistics are averages over 5000 simulations of length 40 (each simulation is HP filtered). The standard deviations across the 5000 simulations are reported in parentheses.

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Figure 5: Putty-Putty Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital, output, investment and capital services using $a = 1.001$ and $b = 1.001$.

Figure 6: Putty-Putty Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital, output, investment and capital services using $a = 2.0$ and $b = 1.001$. 
Figure 7: Putty-Putty Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital, output, investment and capital services using \( a = 3.0 \) and \( b = 1.001 \).

Figure 8: Putty-Putty Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital, output, investment and capital services using \( a = 0.5 \) and \( b = 1.001 \).
Figure 9: Putty-Putty Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital, output, investment and capital services using $a = 0.1$ and $b = 1.001$.

Figure 10: Putty-Putty Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital, output, investment and capital services using $a = 1.001$ and $b = 2.0$. 
Figure 11: Putty-Putty Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital, output, investment and capital services using $a = 1.001$ and $b = 0.5$.

Figure 12: Putty-Putty Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital, output, investment and capital services using $a = 1.001$ and $b = 0.1$. 
Figure 13: Putty-Clay Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital services, output, investment and investment type using $a = 1.001$ and $b = 1.001$.

Figure 14: Putty-Clay Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital services, output, investment and investment type using $a = 2.0$ and $b = 1.001$. 
Figure 15: Putty-Clay Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital services, output, investment and investment type using $a = 3.0$ and $b = 1.001$.

Figure 16: Putty-Clay Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital services, output, investment and investment type using $a = 0.5$ and $b = 1.001$. 
Figure 17: Putty-Clay Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital services, output, investment and investment type using $a = 0.1$ and $b = 1.001$.

Figure 18: Putty-Putty Clay: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital services, output, investment and investment type using $a = 1.001$ and $b = 2.0$. 
Figure 19: Putty-Clay Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital services, output, investment and investment type using $a = 1.001$ and $b = 0.5$.

Figure 20: Putty-Clay Model: Impulse response functions from a 1 standard deviation shock to the price of energy for the price of energy, consumption, wages, labor, energy, capital services, output, investment and investment type using $a = 1.001$ and $b = 0.1$. 