Re-estimating Euler Equations

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Abstract

I estimate an extended version of the incomplete markets consumption model allowing for heterogeneity in discount factors, non-separable preferences for food and other nondurables, liquidity constraints, and precautionary savings. I find statistical support for this model, especially for the non-separability of preferences for food and other nondurables.

JEL D12, D91, E21

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1 Introduction

The estimate of the central parameter of the dynamic macro models, the elasticity of intertemporal substitution (EIS), varies widely from study to study (see Thimme [2016]). I incorporate several extensions: heterogeneity in discount factors, non-separable preferences for food and other nondurables, liquidity constraints, and precautionary savings, and for statistical support for this model.

2 Methodology

Following Dogra and Gorbachev [forthcoming], I derive the household Euler equation from a relatively standard unitary, incomplete markets consumption

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model, using a CRRA utility function. Under rational expectations, an Euler equation between periods $t-2$ and t is given by:

$$(1+r_{h,t})e^{\Delta\theta_{h,t}-2\delta_h}\left(\frac{C_{h,t}}{C_{h,t-2}}\right)^{-\gamma}(1+\lambda_{h,t-2}) = 1+e_{h,t} \qquad (1)$$

where δ_h is the household-specific time discount rate; $C_{h,t}$ non-durable consumption; $\frac{1}{\gamma}$ is the elasticity of intertemporal substitution. Preferences are given by $\theta_{h,t} = \eta_W N^W_{h,t} + \eta_H N^H_{h,t} + \theta' Z_{h,t} + \upsilon_{h,t}$, where $N^W_{h,t}$ and $N^H_{h,t}$ are hours worked by the wife and husband; $w^W_{h,t}, w^H_{h,t}$ are real wages; $Z_{h,t}$ a vector of demographics; and $\upsilon_{h,t}$ an unobservable preference shock. Δ denotes two-year changes in a variable. $(1+r_{h,t}) = (1+r_{h,t-2,t-1})(1+r_{h,t-1,t})$ is the ex-post, marginal tax adjusted, real interest rate on 2-year loans; $\lambda_{h,t-2}$ the normalized Lagrange multiplier on the household's borrowing constraint; and $e_{h,t}$ an expectational error with $\mathbb{E}_{t-2}e_{h,t} = 0$.

Taking logs of both sides, and then a Taylor expansion of (1),

$$\ln(1+e_{h,t}) = e_{h,t} - \frac{1}{2}e^2_{h,t} + R_{h,t} \qquad (2)$$

where $R_{h,t}$ is a remainder containing third and higher order terms. I assume households never receive any news about third and higher order moments: $R_{h,t} = R_h + e^R_{h,t}$, where $\mathbb{E}_{t-2}e^R_{h,t} = 0$. Let $\sigma^2_{h,t-2} = \mathbb{E}_{t-2}[e^2_{h,t}]$ be the year $t-2$ conditional variance of the year t expectational error, and let $\nu_{h,t} = \frac{1}{2}(e^2_{h,t} - \sigma^2_{h,t-2})$ be the household's expectational error concerning $e^2_{h,t}$, which has conditional mean zero.

Attanasio and Low [2004] show that with at least 30 quarters, log-linearized Euler equation estimation is more consistent than a nonlinear GMM estimation. The PSID has a long panel on food consumption (since 1968), but a very short panel on non-durables (since 1999). Thus, I allow for non-separability of food relative to nondurables, and assume that demand for food has the form

$$\ln F_{h,t} = \alpha_0 + \alpha_1 \ln p^F_t + \alpha_2 \ln p^O_t + \beta \ln C_{h,t} + \theta'_F Z_{h,t} + \iota_{h,t} \qquad (3)$$

where p^F_t is the price of food, p^O_t is the price of other non-durables, $Z_{h,t}$ is the vector of demographics, and $\iota_{h,t}$ is an unobservable preference shock. Combining (1), (2) and (3), I obtain the estimating equation:

$$\begin{aligned}\Delta \ln F_{h,t} &= \frac{\beta}{\gamma}\big[\ln(1+r_{h,t}) - 2\delta_h\big] + \alpha_1 \Delta \ln p^F_t + \alpha_2 \Delta \ln p^O_t + \mu \Delta Z_{h,t} \qquad (4)\\ &+ \frac{\beta}{\gamma}\big[\eta_W \Delta N^W_{h,t} + \eta_H \Delta N^H_{h,t} + \ln(1+\lambda_{h,t-2}) + \frac{1}{2}\sigma^2_{h,t-2} - R_h\big] + \varsigma_{h,t}\end{aligned}$$

where $\mu = \frac{\beta}{\gamma}\theta + \theta_F$, and $\varsigma_{h,t} = \dfrac{\beta}{\gamma}(\nu_{h,t} - e_{h,t} - e^R_{h,t} + \Delta\upsilon_{h,t}) + \Delta\iota_{h,t}$.

I assume that measurement error in consumption is stationary and independent of all the regressors, including lagged values of the measurement error and expectations error, consumption levels and interest rates. Thus, the error term $\varsigma_{h,t}$ also contains measurement errors. Their presence further invalidates non-linear GMM estimation of the Euler equation.

3 Estimation

Under assumption of household specific discount factors, δ_h, and the presence of higher order terms, R_h, the standard fixed effects estimator is inconsistent in a dynamic panel data model. I therefore estimate (4) using the Arellano and Bover [1995] two-step system GMM estimator, after removing fixed effects with forward orthogonal transformations.

I use proxy variables to measure $\ln(1 + \lambda_{h,t-2})$ and $\sigma^2_{h,t-2}$. Using proxy variables introduces approximation errors; what is crucial is that these error are uncorrelated with the characteristics of the household s years ago.

According to Carroll [1992], $\sigma^2_{h,t-2}$ appears in the Euler equation because of the precautionary savings motive. I proxy for this motive by including a measure of income uncertainty, which I compute following Gorbachev [2016]. For each year of data, I compute biennial arc growth rates of real household income, then regress them on cohort, age, race, and gender dummies; $\sigma^2_{h,t-2}$ are the squared residuals obtained from these regressions.

I measure liquidity constraints following Dogra and Gorbachev [forthcoming]. First, using direct information from the Survey of Consumer Finances (SCF), I estimate the probability of being denied credit on a set of variables common to both the PSID and the SCF. I then use these parameters to predict the probability of being liquidity constrained in PSID data.

By rational expectations, any variables known at time $t-2$ will be orthogonal to the expectational errors. However, autocorrelation in my sample is present up to the third lag. I use variables dated $t-4$ and $t-5$ as instruments. I limit the number of instruments to two lags and "collapse" these instruments to a single column to reduce the efficiency loss caused by too many instruments. I allow for heteroskedasticity and intra-group correlation, and make the Windmeijer finite-sample correction to standard errors.

4 Results and Conclusions

Table 1 reports estimation results. Column (1) contains the most basic specification. Column (2) allows for a precautionary savings; (3) introduces non-linear effect of liquidity constraints; (4) adds non-separable preferences for food and other non-durables; (5) allows for non-separability with leisure; (6) adds non-separablity with durables; (7) and (8) analyse high and low wealth respondents.

In all specifications the coefficient of interest, $\frac{\beta}{\gamma}$ is statistically significant at the 1% level. Interestingly, as the model becomes more complex, this estimate rises from 0.41 to 0.69.

I find support for non-separability in preferences between food and other nondurables, columns (4) onward. According to the Sargan and Hansen tests, the chosen instruments are valid. First stage regressions (to test for the strength of instruments for $\ln(1 + r_{h,t})$, $\Delta \ln p^F$, and $\Delta \ln p^O$) produce F-statistics of 1588.53, 1154.89, and 1040.74, respectively. Unfortunately, the instruments for $\Pr(\widehat{\text{denied credit}})$ and $(\hat{\sigma}^Y_{h,t})^2$ are very weak (F-stats of less than 5), and are thus not reliable. In columns (7) and (8), I re-estimate the model on households whose wealth holdings were above (or below) the 1984 median (the earliest date for wealth data).[1]

If preferences for food are homothetic ($\beta = 1$), the elasticity of intertemporal substitution, $\frac{1}{\hat{\gamma}}$, ranges between 0.55 and 0.69 for a full sample. For high wealth respondents EIS is 0.6. It is lower but insignificant for low wealth households. If preferences are non-homothetic ($\beta \neq 1$), since food is a necessity, then this coefficient is $\frac{\hat{\beta}}{\hat{\gamma}}$. Using newly available PSID data on non-durables for 2004-2010 period, I follow methodology of Blundell et al. [2008] and estimate $\hat{\beta} = 0.78$, and $\frac{1}{\hat{\gamma}}$ is between 0.70 and 0.88 for full sample, and 0.77 for high wealth subsample.

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[1] Coefficient estimates do not vary with wealth year.

Table 1: *Euler Equation Estimation, 1980 to 2010*

	Full Sample						High Wealth	Low Wealth
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\ln(1+r_{h,t})$	0.408***	0.442***	0.371***	0.550***	0.691***	0.689***	0.602***	0.436
	(0.076)	(0.110)	(0.115)	(0.201)	(0.206)	(0.207)	(0.194)	(0.475)
$(\hat{\sigma}^{Y}_{h,t})^2$		0.054	-0.050	0.068	0.250	0.279	0.070	-0.095
		(0.161)	(0.172)	(0.253)	(0.256)	(0.242)	(0.212)	(0.384)
$\mathrm{Pr}(\widehat{\text{denied credit}})$			0.229	0.000	1.196	1.267		5.024
			(1.089)	(1.189)	(1.229)	(1.267)		(4.661)
$\mathrm{Pr}(\widehat{\text{denied credit}})^2$			0.498	1.208	-4.443	-4.640		-9.722
			(4.413)	(4.874)	(5.099)	(5.170)		(12.914)
$\mathrm{Pr}(\widehat{\text{denied credit}})^3$			-1.335	-2.244	4.193	4.541		5.126
			(4.901)	(5.449)	(5.752)	(5.801)		(11.022)
$\Delta \ln p^O$				0.520*	0.854**	0.784**	0.686**	0.225
				(0.294)	(0.346)	(0.355)	(0.303)	(1.186)
$\Delta \ln p^F$				-0.886	-1.233*	-1.100	-1.164*	0.122
				(0.691)	(0.743)	(0.753)	(0.702)	(2.480)
Age	0.000	0.002	-0.001	0.000	-0.001	-0.002	0.000	-0.029
	(0.002)	(0.002)	(0.003)	(0.003)	(0.004)	(0.004)	(0.002)	(0.020)
Age2	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Δ Adults	0.053	0.160	0.124	0.021	0.194*	0.221**	0.071	0.019
	(0.155)	(0.169)	(0.091)	(0.122)	(0.104)	(0.104)	(0.088)	(0.150)
Δ Kids	0.185	0.097	0.175*	0.225*	0.052	0.020	0.169**	0.088
	(0.132)	(0.154)	(0.099)	(0.117)	(0.109)	(0.106)	(0.071)	(0.182)
Δ Marital Status	0.208	-0.193	-0.175	0.053	-0.445	-0.509	0.315	-0.691
	(0.429)	(0.500)	(0.278)	(0.382)	(0.415)	(0.387)	(0.384)	(0.566)
Δ Hours Worked, Husband					0.199**	0.178**	0.195**	0.030
					(0.086)	(0.088)	(0.095)	(0.171)
Δ Hours Worked, Wife					0.055	0.071	-0.045	0.279
					(0.062)	(0.064)	(0.050)	(0.170)
Δ Home Ownership						-0.148		
						(0.135)		
Observations	87,684	87,684	87,684	87,684	87,684	87,171	64,792	20,468
Number of households	13,910	13,910	13,910	13,910	13,910	13,862	7,974	4,603
Number of Instruments	14	16	26	26	30	34	30	30
F-stat	130	113.3	84.38	67.92	46.78	39.50	59.30	7.040
Prob>F	0	0	0	0	0	0	0	0
Sargan test of overid	8.558	14.16	17.41	16.10	16.10	13.20	18.10	12.62
df	8	9	16	14	16	19	19	16
Prob> χ^2	0.381	0.159	0.421	0.308	0.446	0.815	0.516	0.752
Hansen test of overid	8.372	13.09	16.46	15.94	16.39	13.45	18.29	11.88
df	8	9	16	14	16	19	19	16
Prob> χ^2	0.398	0.117	0.359	0.317	0.426	0.828	0.503	0.700
Joint Significance Test p-value			0.504	0.770	0.681	0.761		0.441

Notes: Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

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